Integrating Bottom-Up into Top-Down: A Mixed Complementarity Approach

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Modeling Energy-Economy Interactions: Five Approaches, edited by Charles Hitch. Published by Resources for the Future

"Energy-Economy Interactions: The Fable of the Elephant and the Rabbit?" by William Hogan and Alan S. Manne.

1977

Motivation

- In many energy policy studies, the energy sector is appropriately viewed in isolation from the remainder of the economy.
- In some situations this may be inappropriate, as there may be *two way interdependence* between energy markets and the rest of the economy.
- Even a large change in energy markets may represent a small fraction of aggregate economic output.
- There may be virtual one-way linkages: growth in aggregate GDP influence energy demand, but not vice versa.
- If, however, two-way linkages are important, then the analysis of energy market issues demands an economy-wide perspective.

The Elephant and the Rabbit

- The energy value share of GDP is typically on the order of 4-5% in industrial countries.
- This is something like *elephant-rabbit stew*. If such a recipe contains just one rabbit (the energy sector) and one elephant (the rest of the economy), doesn't it still tast very much like elephant stew?
- But what if energy prices double, triple or quadruple, and there is sufficient time for the economy to respond? How much will this cost the rest of the economy?
- For large reductions in energy use, the value share of energy in aggregate output need not remain fixed. If the value share rises, the metaphor of the elephant and the rabbit may no longer be apprpriate.

Overview

Impact Assessment of Energy Policies

- Motivation
- Mixed Complementarity
- From Bottom-up to Top-Down
- Illustration
- Conclusion

Complementary (hybrid) modeling framework:

- Comprehensive coverage of markets:
  - interactions, distortions, imperfections
- Incorporation of income flows:
  - origination and spending of income (endowments and preferences)

Top-down (general equilibrium)

- Technological foundation:
  - discrete technological options

Bottom-up (partial equilibrium)

Synthesis of Top-down and Bottom-up

Dichotomy of Top-down and Bottom-Up

Policy focus and availability of solvers ⇒ mathematical format

- **Top-down:** system of equations

  \+ equilibrium constraints in prices and quantities

  – activity analysis, weak inequalities

- **Bottom-up:** mathematical programs

  \+ activity analysis, weak inequalities

  – integrability restrictions

Excursus: Integrability

- Equivalence of first order conditions with equilibrium conditions:

  - coincidence of shadow price of mathematical programming constraints with market prices

  - restrictive symmetry and efficiency properties of mathematical programs:
    - symmetry of (cross-price) demand elasticities
    - omission of multiple agents (income effects)
    - efficient allocation <==> taxes, price caps, spillover externalities

  - sophisticated sequential joint maximization (SJM) techniques to overcome „non-integrabilities“ in optimization approach

Framework for Synthesis: Mixed Complementarity Problem (MCP) Format (Rutherford 1995, JEDC)

Mixed Complementarity Problem (MCP):

$$\begin{aligned}
&Given: && f : R^N \to R^N, \quad l, u \in R^N \\
&Find: && z, w, v \in R^N \\
&s.t.: && F(z) - w + v = 0 \\
& && l \le z \le u, \quad w \ge 0, \quad v \ge 0, \\
& && w^T (z - l) = 0, \quad v^T (u - z) = 0
\end{aligned}$$

Mixed: Mixture of equalities and inequalities

Complementarity: Complementarity between system variables and system conditions

+ coverage of system of equations and mathematical programs as subcases
+ equilibrium constraints in prices and quantities (no integrability restrictions)
+ activity analysis, weak inequalities
+ availability of large-scale robust solvers (PATH)

The Arrow-Debreu-Model as MCP

p := a non-negative n-vector of prices for all goods and factors ($I=\{1,...,n\}$)

y := a non-negative m-vector of activity levels for CRTS production sectors ($J=\{1,...,m\}$)

M := a non-negative k-vector of incomes ($H=\{1,...,k\}$)

Zero profit condition for CRTS producers:

$$-\Pi_j(p) = C_j(p) - R_j(p) \geq 0 \quad \forall j$$

Market clearance for all goods and factors:

$$\sum_j y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h b_{ih} \geq \sum_h d_{ih} \quad \forall i$$

Budget constraints for households:

$$\sum_h p_i b_{ih} = M_h \geq \sum_h p_i d_{ih} \quad \forall h \qquad d_{ih}(p, M_h) \equiv \arg\max \left\{ U_h(x) \,\middle|\, \sum_i p_i x_i = M_h \right\}$$

Complementarity Features of Economic Equilibria

- Motivation
- **Mixed Complementarity**
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Walras‘ law („Non-satiation“) yields:

$$\sum_j y_j \Pi_j(p) = 0 \qquad \text{resp.} \qquad y_j \Pi_j(p) = 0 \quad \forall j$$

$$p_i \left(\sum_j y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h b_{ih} - \sum_h d_{ih} \right) = p_{i\xi} \xi_i = 0 \quad \forall i$$

$$M_h \left(\sum_h p_i b_{ih} - \sum_h p_i d_{ih} \right) = 0 \quad \forall h$$

Ergo: The problem of solving the economic equilibrium corresponds to a MCP where:

$$z = [y, p, M] \qquad \text{resp.} \qquad f(z) = \left[\Pi_j(p), \xi_i, \left(\sum_h p_i b_{ih} - \sum_h p_i d_{ih} \right) \right]$$

Economic Equilibrium Problem as MCP

Equivalence of market equilibrium problem with complementarity problem:

$$Given: f: R^n \to R^n$$

$$Find: z \in R^n$$

$$subject\,to: f(z) \geq 0,\ z \geq 0,\ z^T f(z) = 0$$

$$l = 0,\ u = +\infty, \quad z = [y, p, M],\ f(z) = \left[\Pi_j(p), \xi_i, \left(\sum_h p_i b_{ih} - \sum_h p_i d_{ih} \right) \right]$$

Likewise: Mathematical Programs as a special case of MCP!

From Top-down towards Bottom-up:

- write equations as weak inequalities
- specify complementarity
- add activity analysis/weak inequalities for energy sectors (replacing smooth production function representation)

From Bottom-up towards Top-down:

- re-cast NLP as an MCP
- add multiple markets
- add income constraints

The 2x2x1 - Model

Equilibrium conditions for competitive 2x2x1-economy:

Zero profit:	$p_i = r K_i^y \left(r,w\right) + w L_i^y \left(r,w\right)$	$i=1,2$
Capital demand:	$K_i = K_i^y\left(r,w\right) Y_i = \frac{\partial p_i}{\partial r} Y_i$	$i=1,2$
Labor demand:	$L_i = L_i^y\left(r,w\right) Y_i = \frac{\partial p_i}{\partial w} Y_i$	$i=1,2$
Market clearance:	$Y_i = X_i$	$i=1,2$
Goods markets:	$X_i = X_i\left(p_1, p_2, M\right)$	$i=1,2$
Capital market:	$\sum_{i=1}^{2} K_i^y\left(r,w\right) Y_i = \bar{K}$	
Income definition:	$M = r\bar{K} + w\bar{L}$	
Numéraire:	$w=1$	

➡ **System of 12 nonlinear equations in 12 variables**

N.B.: implicit variables ⇒ K_i, L_i, X_i, M

Coefficient Form versus Calibrated Share Form

- Motivation
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	CES coefficient form:	CES calibrated share form:
Production:	$y = \gamma \left(\sum_i \alpha_i x_i^\rho \right)^{1/\rho}$	$y = \bar{y} \cdot \left[\sum_i \left(\theta_i \cdot \left(\frac{x_i}{\bar{x}_i} \right)^\rho \right) \right]^{1/\rho}$
Cost:	$C = \gamma^{-1/\sigma} \left[\sum_i \alpha_i^\sigma \cdot \gamma^{(\sigma-1)\cdot\rho} \cdot w_i^{1-\sigma} \right]^{1/(1-\sigma)} \cdot y$	$C = \bar{C} \cdot \left[\sum_i \theta_i \cdot \left(\frac{w_i}{\bar{w}_i} \right)^{1-\sigma} \right]^{1/(1-\sigma)} \cdot \frac{y}{\bar{y}}$
Demand:	$x_i = \gamma^{\sigma-1} \cdot \left(\frac{\alpha_i p}{w_i} \right)^\sigma \cdot y$	$x_i = \bar{x}_i \cdot \frac{y}{\bar{y}} \cdot \left(\frac{c}{\bar{c}} \cdot \frac{\bar{w}_i}{w_i} \right)^\sigma$

Advantage of *calibrated share form*:

No messy inverting:

➡ Direct calibration from benchmark values

Calibration - The Basics

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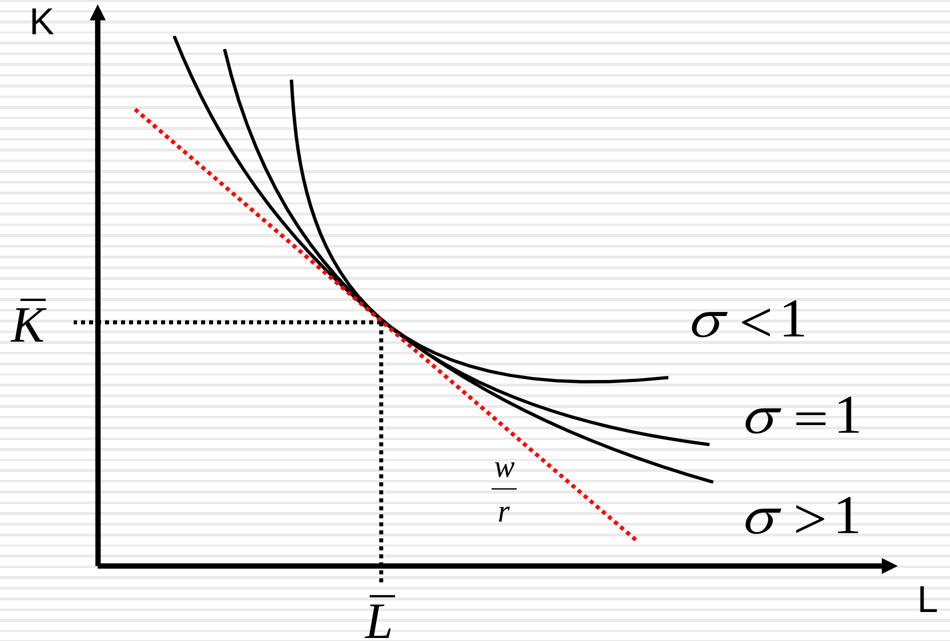
CES function is determined by:

- Quantities (Zeroth order approximation - anchor point)
- Prices (First order approximation - slope)
- Elasticity (Second order approximation - curvature)



Calibration - Microconsistent Dataset

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Benchmark equilibrium:

Price convention: $p_1 = p_2 = r = w = 1$

	Y_1	Y_2	Household	$\sum$
Y_1	40	–	-40	0
Y_2	–	40	-40	0
$\overline{K}$	-20	-30	50	0
$\overline{L}$	-20	-10	30	0
$\sum$	0	0	0	

- Zero profit: column sum
- Market clearance: row sum

} input-output table

- Budget constraint

→ Social Accounting Matrix (SAM)

MCP-Implementation of 2x2x1 - Model

- Motivation
- **Mixed Complementarity**
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Equilibrium conditions	Variables	Complementarity features
Zero profit	**Activity variables**	
$r^{0.5}w^{0.5} \geq p_1$	$y_1 \geq 0$	$\left(r^{0.5}w^{0.5} - p_1\right)y_1 = 0$
$r^{0.75}w^{0.25} \geq p_2$	$y_2 \geq 0$	$\left(r^{0.75}w^{0.25} - p_2\right)y_2 = 0$
Market clearance	**Price variable**	
$40\,y_1 \geq 40\frac{M}{80}\frac{1}{p_1}$	$p_1 \geq 0$	$\left(40\,y_1 - 40\frac{M}{80}\frac{1}{p_1}\right)p_1 = 0$
$40\,y_2 \geq 40\frac{M}{80}\frac{1}{p_2}$	$p_2 \geq 0$	$\left(40\,y_2 - 40\frac{M}{80}\frac{1}{p_2}\right)p_2 = 0$
$30 \geq 20\,y_1\frac{p_1}{w} + 10\,y_2\frac{p_2}{w}$	$w \geq 0$	$\left(30 - \left(20\,y_1\frac{p_1}{w} + 10\,y_2\frac{p_2}{w}\right)\right)w = 0$
$50 \geq 20\,y_1\frac{p_1}{r} + 30\,y_2\frac{p_2}{r}$	$r \geq 0$	$\left(50 - \left(20\,y_1\frac{p_1}{r} + 30\,y_2\frac{p_2}{r}\right)\right)r = 0$
Budget constraint	**Income variable**	
$30w + 50r \geq M$	$M \geq 0$	$\left((30w + 50r) - M\right)M = 0$

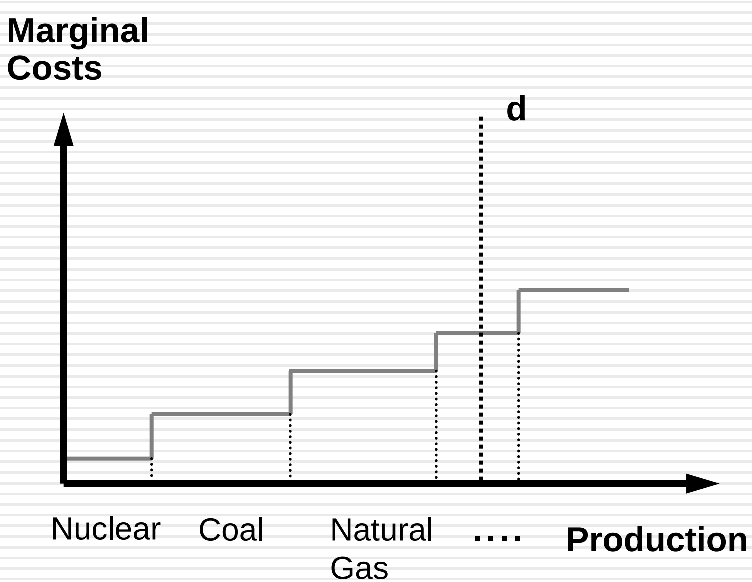
From Bottom-Up to Top-Down (1)

- Motivation
- Mixed Complementarity
- ➤ **From Bottom-up to Top-Down**
- Illustration
- Conclusion

Least-cost energy supply planning problem:

$$\min \quad \sum_i c_i x_i$$
$$s.t. \quad \sum_i x_i \geq d \qquad \perp p_E$$
$$a_i x_i \leq b_i \qquad \perp r_i$$

x_i := activity level of technology i,
c_i := unit cost coefficient (Leontief) of technology i,
a_i := unit capacity requirement (Leontief) of technology i,
b_i := capacity constraint for technology i,
d := exogenous energy demand
p_E := shadow price of energy market constraint
r_i := shadow price of capacity constraint for technology i



From Bottom-Up to Top-Down (2)

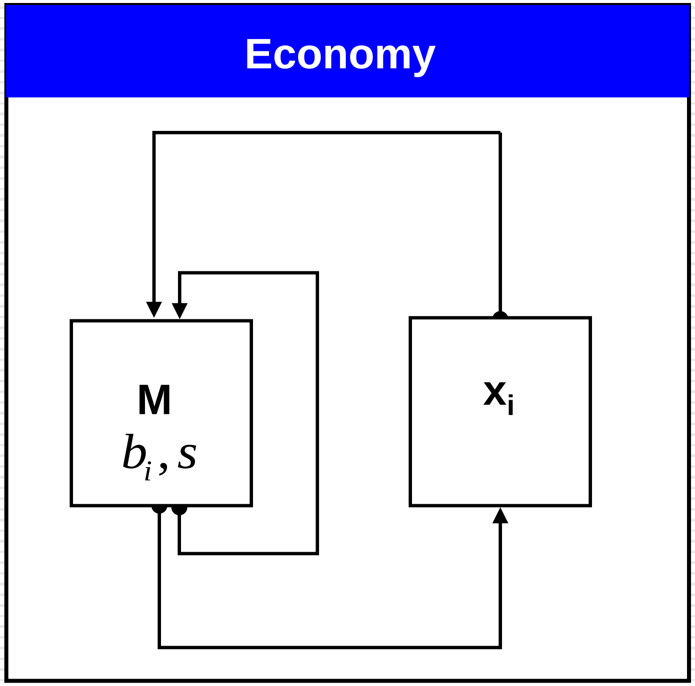
MCP formulation of supply planning problem:

Equilibrium conditions	Variables	Complementarity features
Zero profit	**Activity variable**	
$c_i + a_i r_i \geq p_E,$	$x_i \geq 0$	$x_i \left(c_i + a_i r_i - p_E\right) = 0$
Market clearance	**Price variable**	
$\sum_i x_i \geq d$	$p_E \geq 0$	$p_E \left(\sum_i x_i - d\right) = 0$
$a_i x_i \leq b_i$	$r_i \geq 0$	$r_i \left(a_i x_i - b\right) = 0$

From Bottom-Up to Top-Down (3)

Simplistic CGE extension:

- additional macro-good as endowment (input to energy production and final consumption)
- only energy production activities
- Cobb-Douglas preferences in energy and the macro-good



From Bottom-Up to Top-Down (4)

- Motivation
- Mixed Complementarity
- From Bottom-up to Top-Down
- Illustration
- Conclusion

MCP formulation of simplistic CGE-extension:

Equilibrium conditions	Variables	Complementarity features
Zero profit	**Activity variable**	
$c_i p + a_i r_i \geq p_E,$	$x_i \geq 0$	$x_i \left(c_i p + a_i r_i - p_E \right) = 0$
Market clearance	**Price variable**	
$\sum_i x_i \geq \alpha M / p_E$	$p_E \geq 0$	$p_E \left(\sum_i x_i - \alpha M / p_E \right) = 0$
$a_i x_i \leq b_i$	$r_i \geq 0$	$r_i \left(a_i x_i - b \right) = 0$
$\sum_i c_i x_i + (1-\alpha) M / p \leq s$	$p \geq 0$	$p \left(\sum_i c_i x_i + (1-\alpha) M / p - s \right) = 0$
Budget constraint	**Income variable**	
$sp + \sum_i r_i b_i \geq M$	$M \geq 0$	$M \left(sp + \sum_i r_i b_i - M \right) = 0$

p := market price of the macro-good,

M := income of the representative agent,

s := endowment with macro-good,

α := share parameter for energy in Cobb-Douglas utility function

Benchmark Data of Stylized Economy

(Böhringer & Rutherford 2008, ENEECO)

Table 1: Base Year Equilibrium

	ROI	COA	GAS	OIL	ELE	RA
ROI	200	-5	-5	-10	-10	-170
COA		15			-15	
GAS			15		-15	
OIL				30		-30
ELE	-10				60	-50
Capital	-80				-20	100
Labor	-110	-5	-5	-10		130
Rent		-5	-5	-10		20

Key	
ROI:	rest of industry
COA:	coal
GAS:	gas
OIL:	oil
ELE:	electricity
RA:	household

Embodied least-cost energy supply problem:

$$\min \sum_i \sum_t p_i a_{ijt} y_{it}$$

s.t. $$\sum_t y_{jt} + \sum_{i \neq j} a_{ji} \bar{y}_i + \sum_h w_{jh} \geq \sum_h \bar{d}_{jh}$$

$$y_{jt} \leq \sum_h w_{hjt}$$

Here:

Supply of demand for energy good j (electricity) by alternative technologies t subject to capacity constraints!

Technologies for Electricity Generation

- Motivation
- Mixed Complementarity
- From Bottom-up to Top-Down
- ➢ **Illustration**
- Conclusion

Table 2: Cost Structure of Active Technologies (Base Year)

	coal	gas	nuclear	hydro
ELE	20	20	12	8
ROI	-1	-1	-8	
GAS		-15		
COA	-15			
Capital	-4	-4	-4	-8

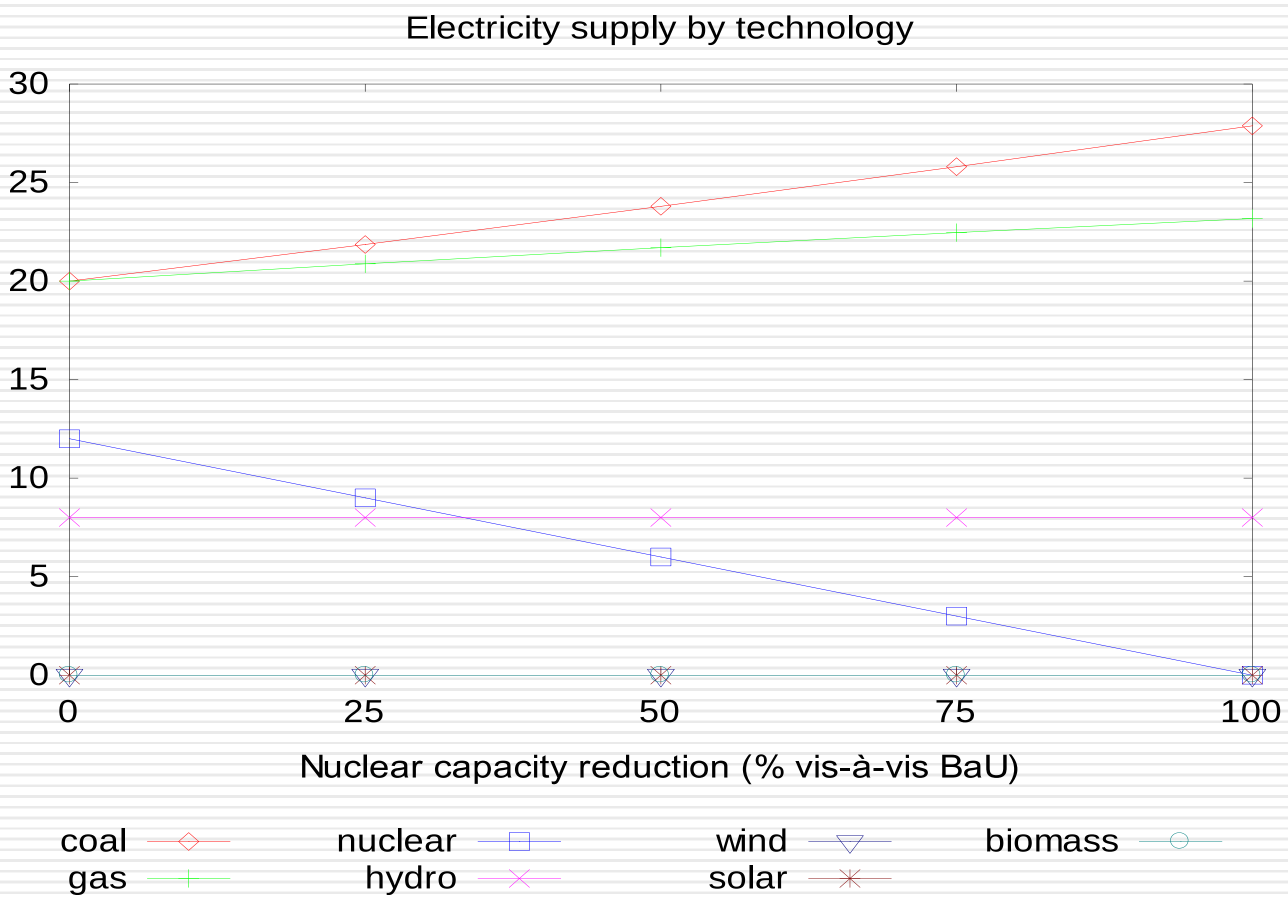
Table 3: Cost Structure of Inactive Technologies (Base Year)

	wind	solar	biomass
ELE	1	1	1
ROI	-0.2	-0.3	-0.4
Capital	-0.9	-0.8	-0.7
wind	-1		
sun		-1	
trees			-1

Policy Simulation: Nuclear Phase-Out

- Motivation
- Mixed Complementarity
- From Bottom-up to Top-Down
- ➤ Illustration
- Conclusion

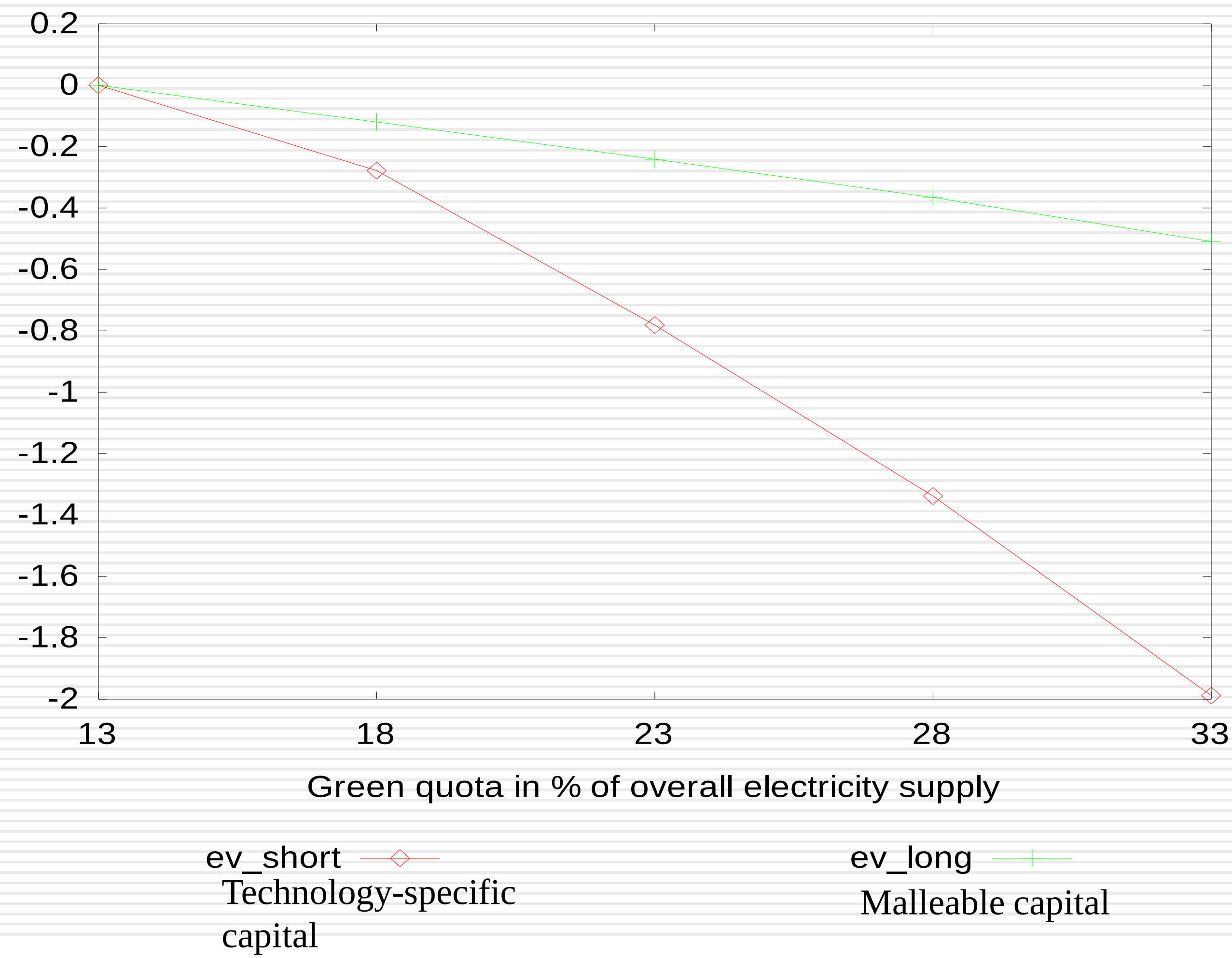
Gradual reduction in permissible nuclear power capacity:



Policy Simulation: Green Quota

- Motivation
- Mixed Complementarity
- From Bottom-up to Top-Down
- **Illustration**
- Conclusion

Subsidized increased of renewable electricity production:



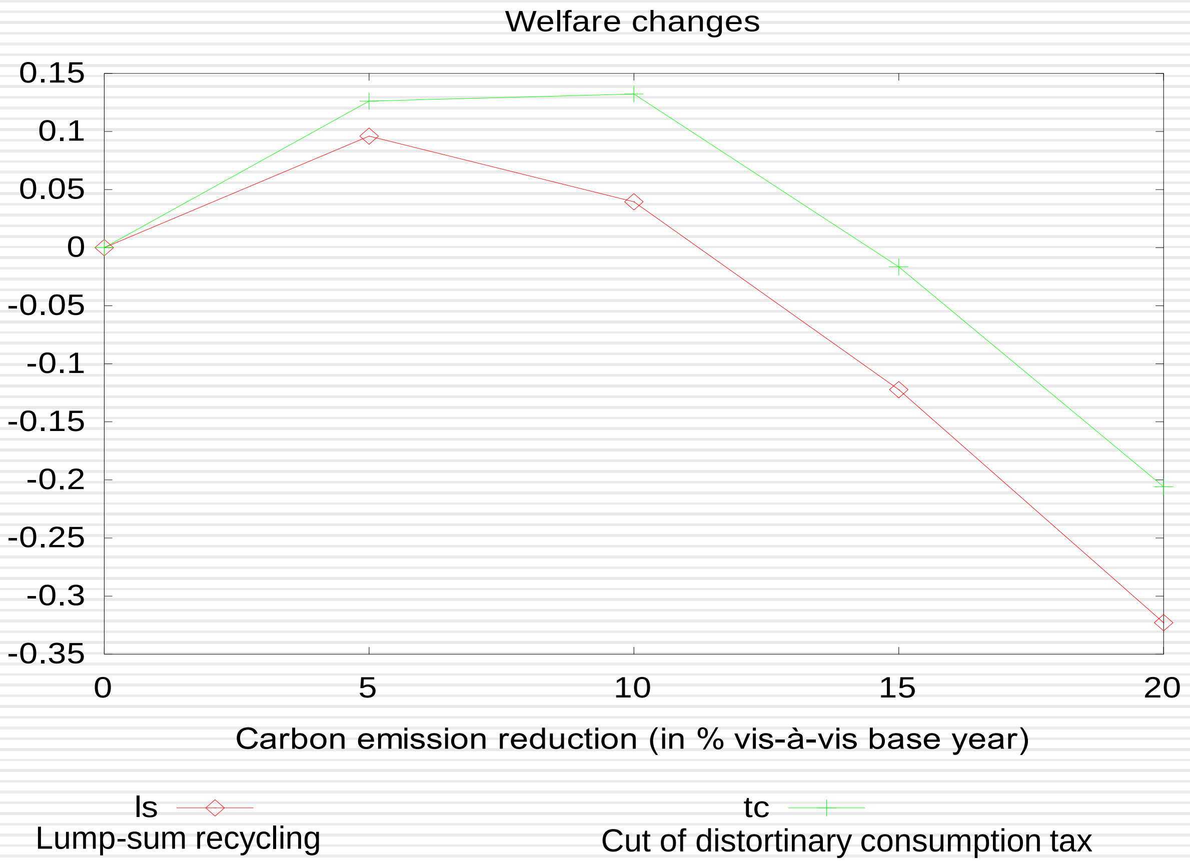
Policy Simulation: Environmental Tax Reform

- Motivation
- Mixed Complementarity
- From Bottom-up to Top-Down
- **Illustration**
- Conclusion

Imposition and recycling of carbon taxes:

– initial partial consumption tax on non-energy commodities

– fixed level of public good provision



Summary

- **Perceived Dichotomy: Bottom-up versus Top-Down**
  - special (restricted) cases of general equilibrium conditions
  - policy focus and availability of efficient/robust algorithms

- **MCP framework for synthesis (hybrid models) :**
  - economic richness of top-down (CGE) models
  - technological foundation of bottom-up models
  - availability of solution algorithms for “large-scale” problems

Variation: Decomposition of Large-Scale Hybrid Models

(Böhringer & Rutherford 2009, JEDC)

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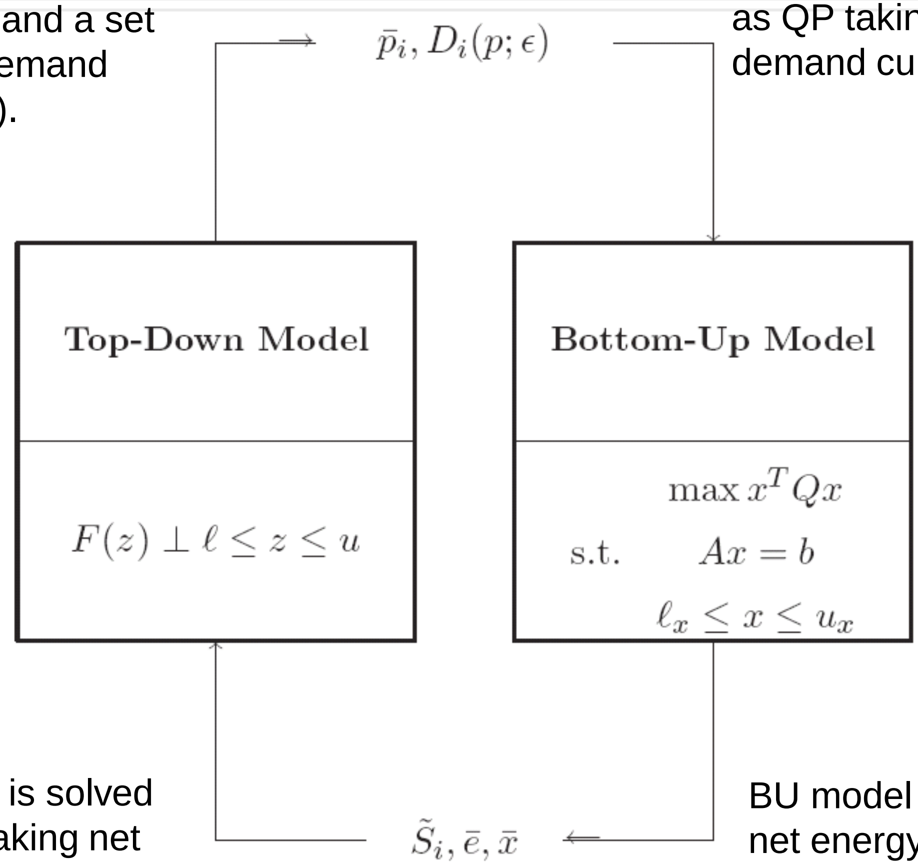
TD model determines prices (p_i) and a set of linear demand curves (D_i).

BU model is solved as QP taking prices and demand curves as given.



TD model is solved as MCP taking net energy supplies (e_i) and energy sector inputs (x) as given.

BU model determines net energy supplies and energy sector inputs.

Outlook: Application To Energy Policy Scenarios

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- From Bottom-up to Top-Down
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