Activity Analysis:

From NLP To MCP

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Linear and Isoelastic (Demand) Supply Functions

1. Linear models:

$$D(p) = \bar{d}\left(1 - \epsilon\left(\frac{p}{\bar{p}} - 1\right)\right)$$

$$S(p) = \bar{s}\left(1 + \eta\left(\frac{p}{\bar{p}} - 1\right)\right)$$

2. Isoelastic models:

$$D(p) = \bar{d}\left(\frac{p}{\bar{p}}\right)^{-\epsilon}$$

$$S(p) = \bar{s}\left(\frac{p}{\bar{p}}\right)^{\eta}$$

Addendum: Calibrated linear demand function

Demand function based on observed price/quantity pair ($\bar{p}$, $\bar{q}$) and (negative) elasticity ε

$$q(p) = \bar{q}\,(1 + \varepsilon(\frac{p}{\bar{p}} - 1))$$

Conventional linear demand function:

$$q(p) = a - bp \quad (*)$$

Observation benchmark market demand:

$$\bar{q} = a - b\bar{p} \qquad (1)$$

Price elasticity of demand at benchmark point:

$$\varepsilon = \frac{dq}{dp}\frac{\bar{p}}{\bar{q}} = -b\frac{\bar{p}}{\bar{q}} \qquad (2)$$

(1) ➔ (1'): $a = \bar{q} + b\bar{p}$; (1) ➔ (2'): $-b = \varepsilon\frac{\bar{q}}{\bar{p}}$

(1'), (2') in (*): $q = \bar{q} - \varepsilon\frac{\bar{q}}{\bar{p}}(\bar{p} - p) = \bar{q}\,(1 + \varepsilon(\frac{p}{\bar{p}} - 1))$

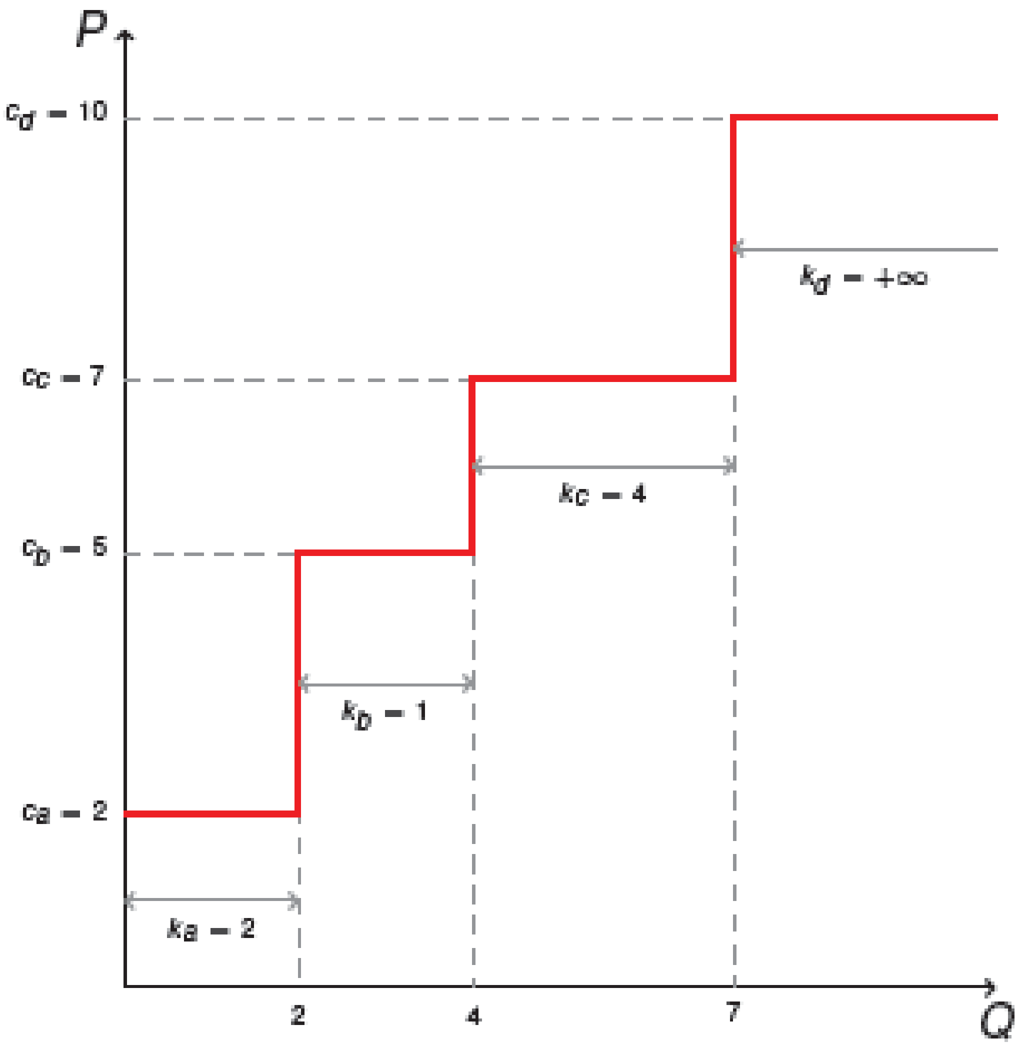
Activity Analysis

When there are a discrete set of production technologies, each characterized by a marginal cost and a capacity, the supply curve becomes a step function corresponding to the sorted sequence of plant capacities.

Consider a market in which the commodity is supply by the following four technologies:

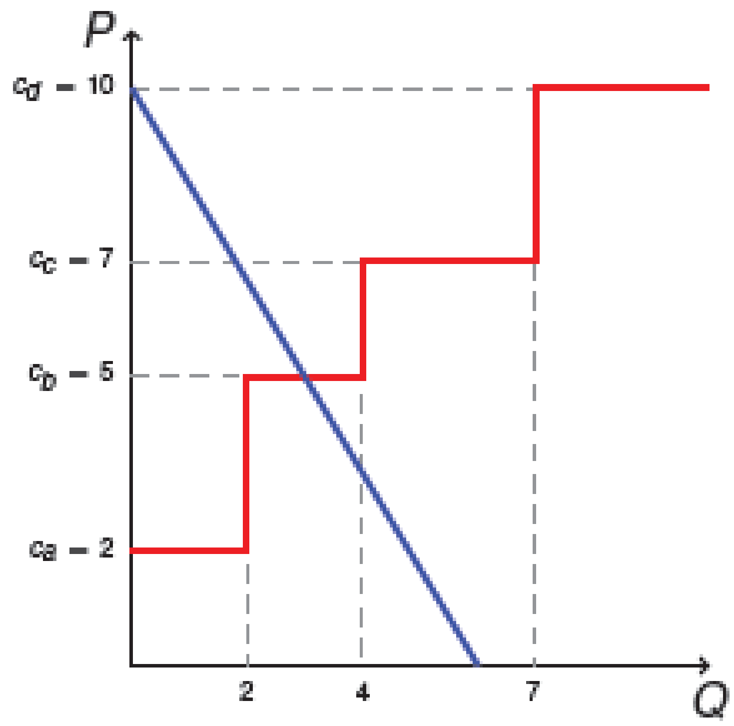
	c_j	k_j
a	2	2
b	5	2
c	7	4
d	10	∞

Activity Analysis Supply Curve



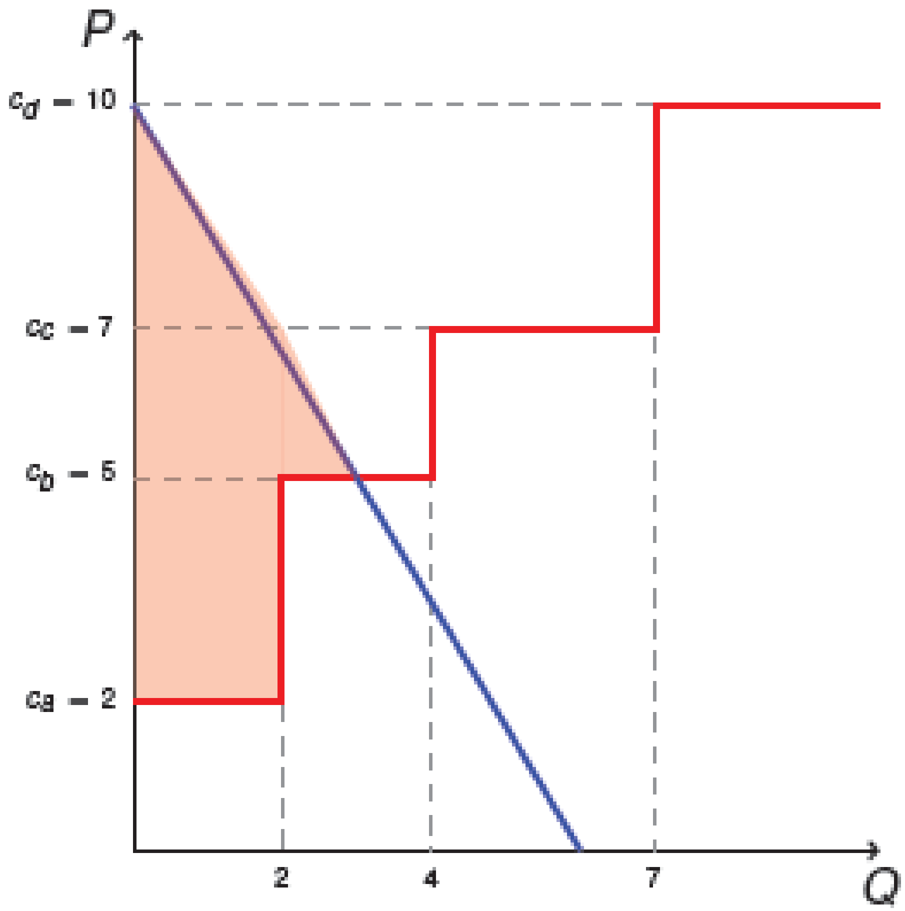
Market Equilibrium with Activity Analysis

Consider a market equilibrium when there are multiple discrete supply technologies. As in the conventional continuous Marshallian model, the equilibrium price and quantity is defined by the intersection of the supply and demand schedules:



Market Equilibrium and Social Surplus

A convenient property of the competitive market allocation is that it *maximizes* social surplus, as illustrated in this figure:



Constrained Optimization Approach

Let $Q_t \geq 0$ denote output from technology t, P denote the equilibrium price, PS and CS denote producer and consumer surplus. The market equilibrium then solves:

$$\max PS + CS$$

subject to:

- Market supply equals technology output:

$$S = \sum_t Q_t$$

- Market equilibrum price is on the demand curve:

$$P = 10 - \frac{5}{6}S$$

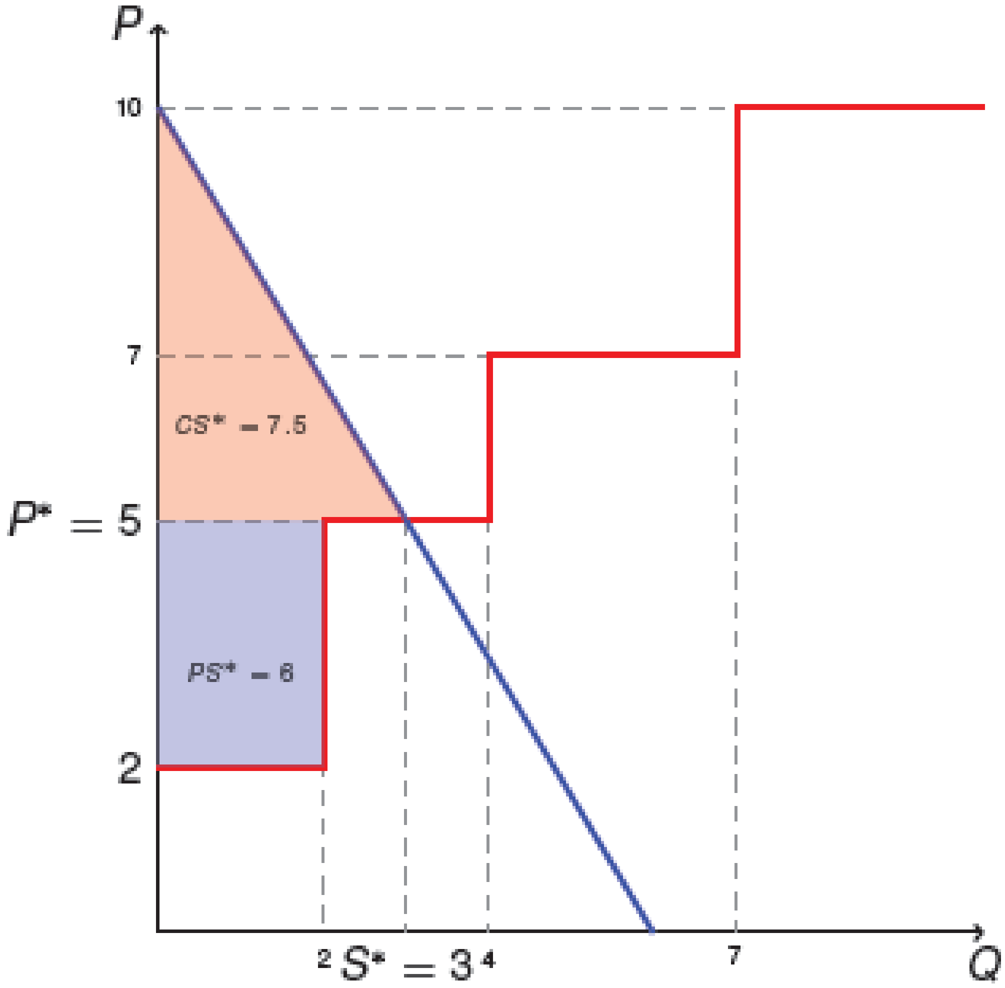
- Producer surplus is the area below the market price and above the cost of production:

$$PS = \sum_t (P - c_t)Q_t$$

- Consumer surplus is the area under the demand curve:

$$CS = \frac{(10 - P)}{2}S$$

Geometric Interpretation of the Equilibrium



GAMS Code – Sets and Data

```
$title  surplus maximization and market equilibrium

set     t /a,b,c,d/;

table   tech     Technology
        cost     cap
a       2        2
b       5        2
c       7        4
d       10       inf;

parameter        c(t)     Cost by technology;
c(t) = tech(t,"cost");
```

GAMS Code – Variable Declaration

```
nonnegative variables   P,PS,CS,s,Q(t);

free variable           obj;

equations        price, supply, psurplus, csurplus, objective;

price..          P =e= 10 - S*10/6;

supply..         S =e= sum(t, Q(t));

psurplus..       PS =e= sum(t, (P-c(t))* Q(t));

csurplus..       CS =e= (10 - P)*S/2;

objective..      OBJ =e= CS + PS;

Q.UP(t) = tech(t,"cap");

model equil /all/;
solve equil using nlp maximizing OBJ;
```

GAMS Listing File

```
                    LOWER          LEVEL          UPPER        MARGINAL
---- VAR P            .            5.0000         +INF            .
---- VAR PS           .            6.0000         +INF            .
---- VAR CS           .            7.5000         +INF            .

---- VAR Q
a                     .            2.0000         2.0000        3.0000
b                     .            1.0000         2.0000        EPS
c                     .              .            4.0000       -2.0000
d                     .              .            +INF         -5.0000

---- VAR obj        -INF           13.5000        +INF            .
```

Hands-on session:

1. Reformulate the NLP model as an economic MCP problem
2. Insert emission accounting to technology a and impose an emission cap at the level of 50% of the reference emissions